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# Categorical Automata Learning Framework

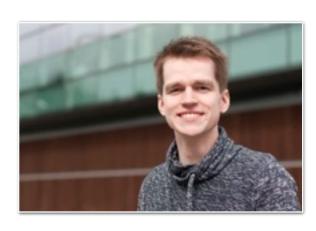
Alexandra Silva University College London



Matteo Sammartino **UCL** 



Gerco van Heerdt **UCL** 

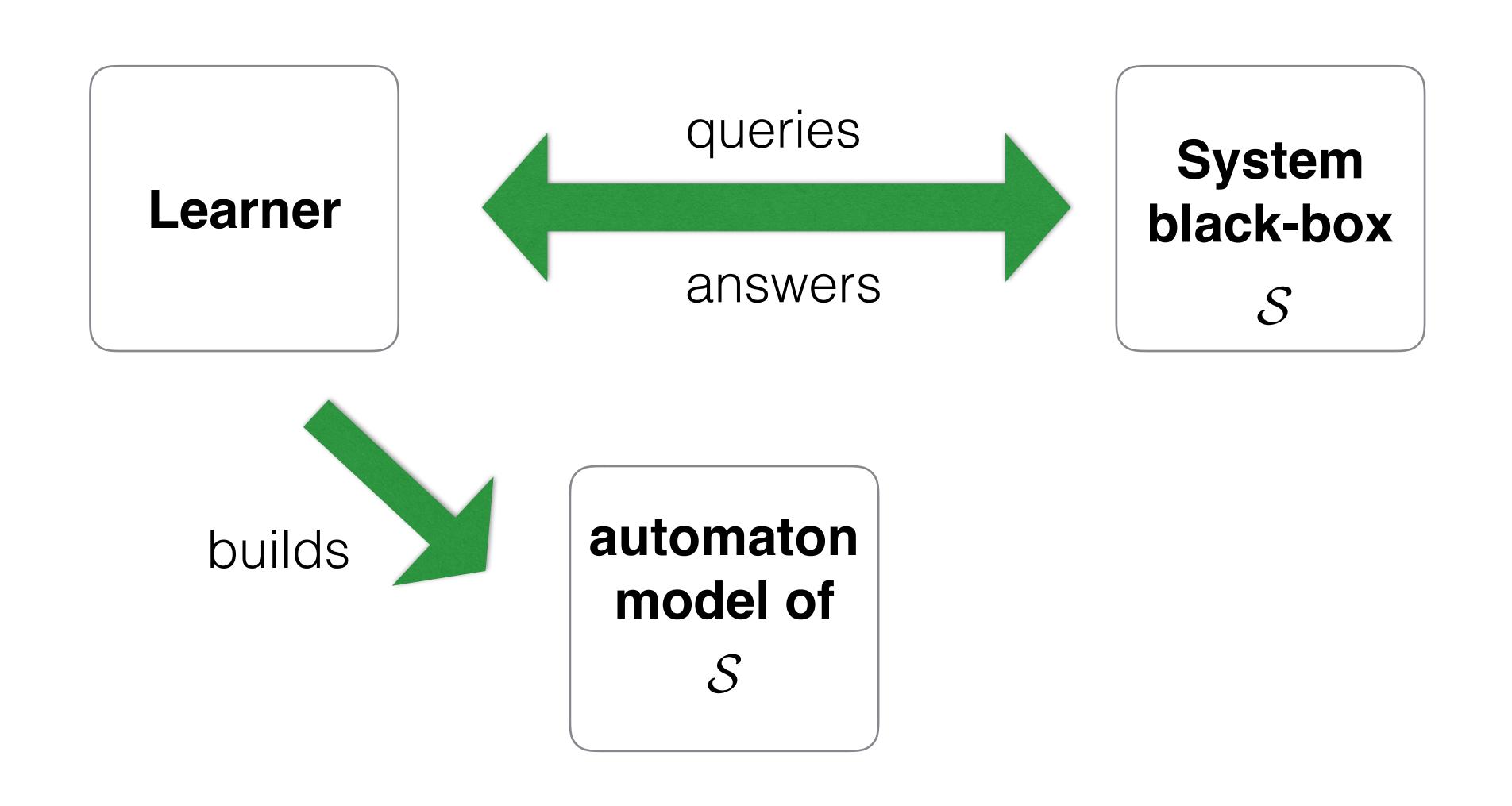


Joshua Moerman Radboud University

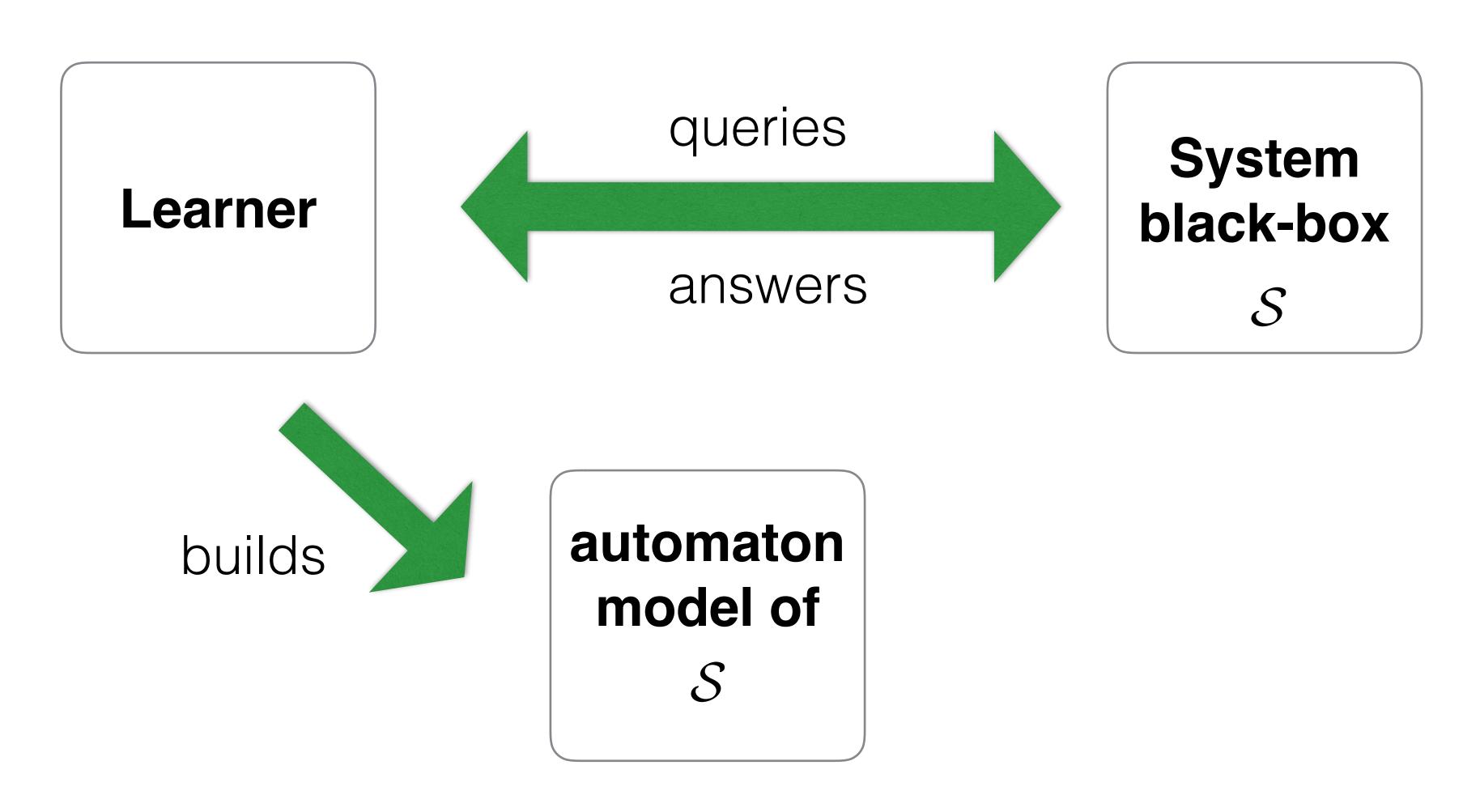


Maverick Chardet ENS Lyon

# Automata learning



# Automata learning



No formal specification available? Learn it!

Finite alphabet of system's actions A set of system behaviors is a regular language  $\mathcal{L} \subseteq A^\star$ 

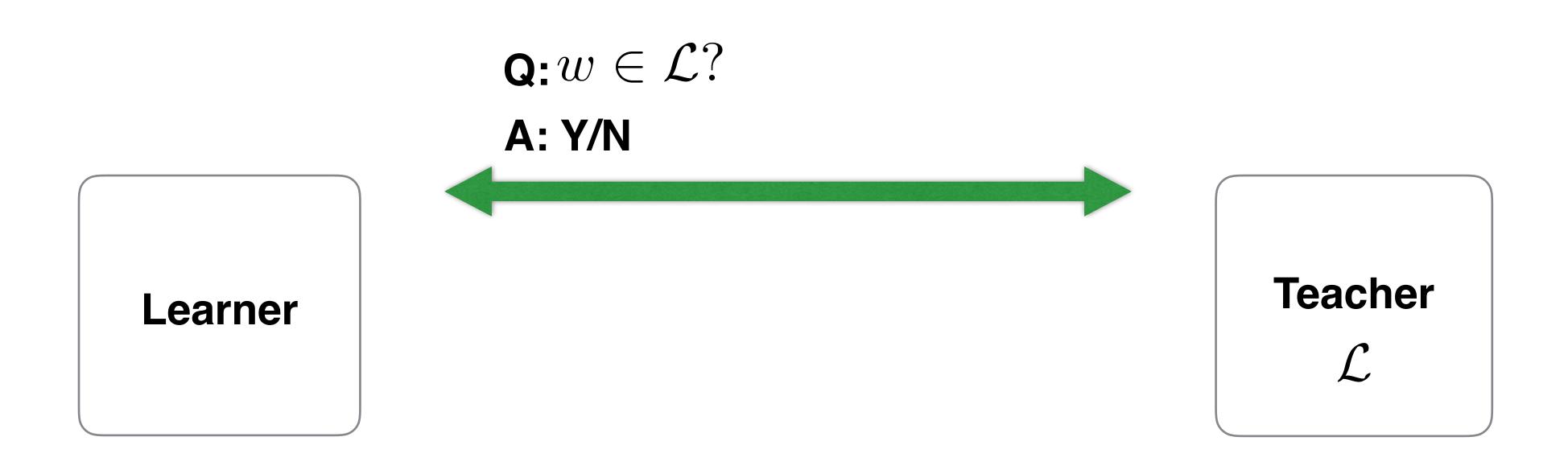
Finite alphabet of system's actions A set of system behaviors is a regular language  $\mathcal{L} \subseteq A^{\star}$ 

Learner

**Teacher** 

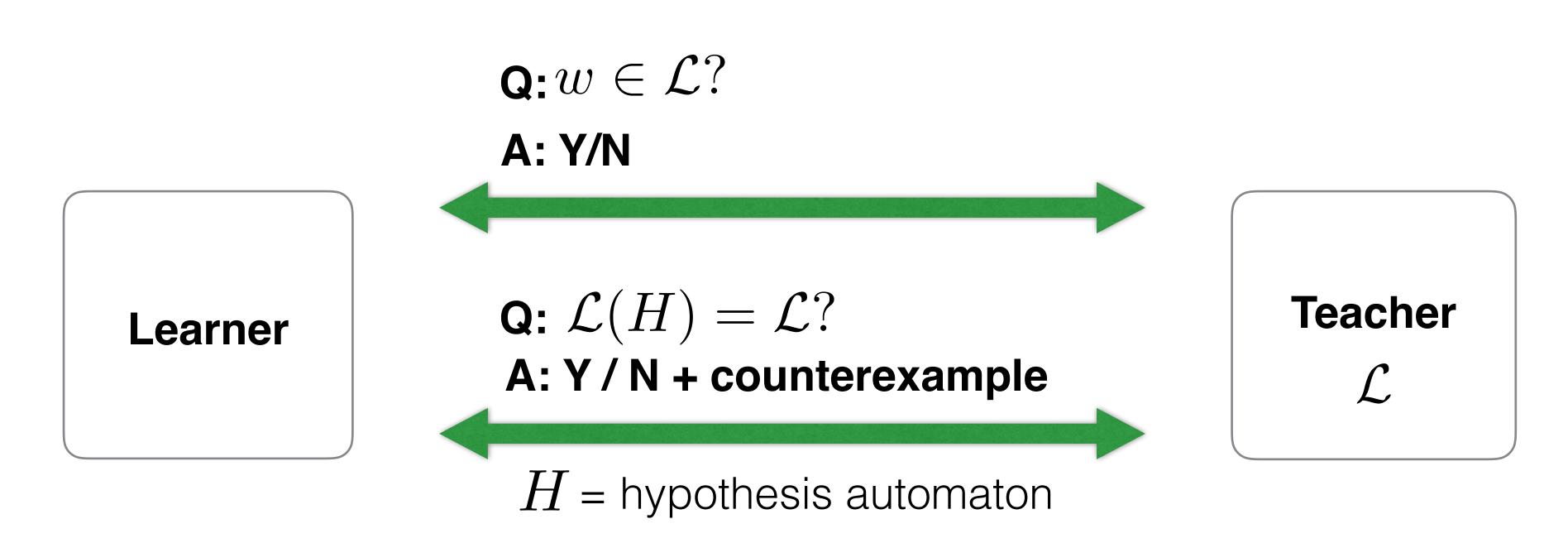
 $\mathcal{L}$ 

Finite alphabet of system's actions A set of system behaviors is a regular language  $\mathcal{L} \subseteq A^{\star}$ 



Finite alphabet of system's actions A

set of system behaviors is a **regular language**  $\mathcal{L} \subseteq A^*$ 



Finite alphabet of system's actions A

set of system behaviors is a **regular language**  $\mathcal{L} \subseteq A^*$ 

 $\mathbf{Q}: w \in \mathcal{L}$ ?

A: Y/N

Learner

Q:  $\mathcal{L}(H) = \mathcal{L}$ ?

A: Y / N + counterexample

H = hypothesis automaton

builds

Minimal DFA accepting  $\mathcal{L}$ 

**Teacher** 

 $\mathcal{L}$ 

## A zoo of automata

Probabilistic

Weighted

Non-deterministic

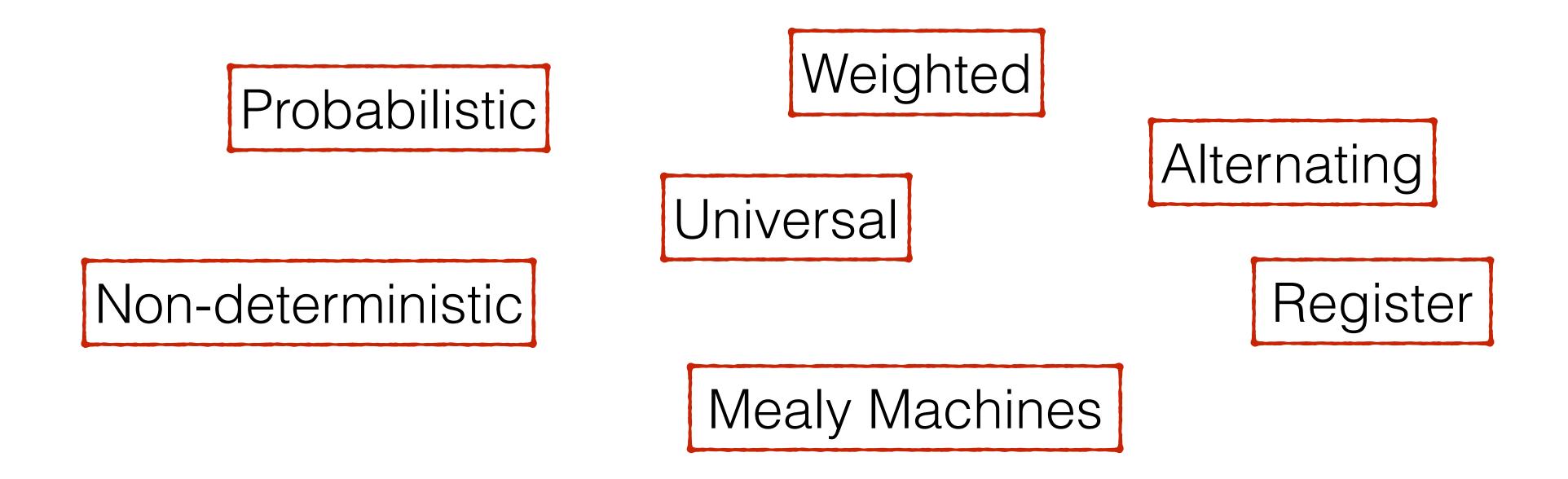
Universal

Alternating

Register

Mealy Machines

## A zoo of automata

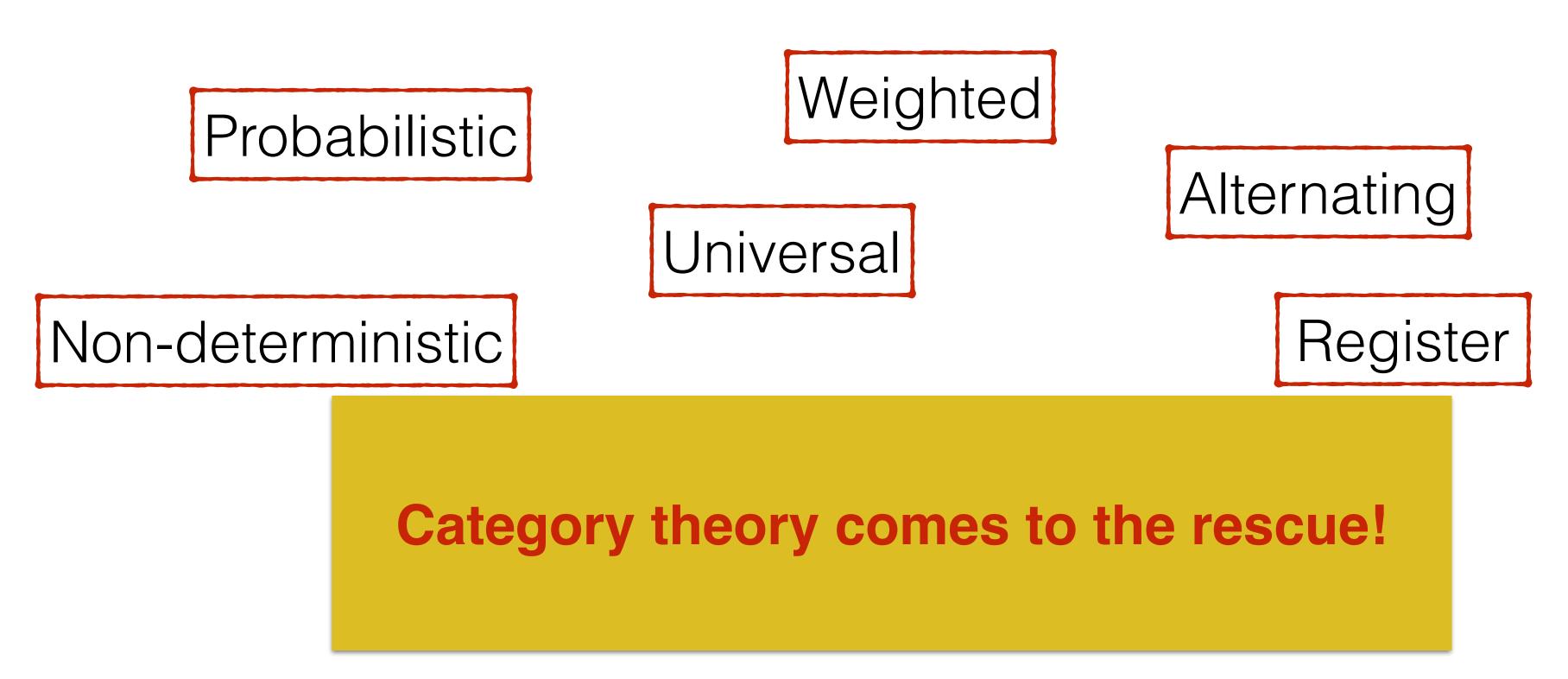


Algorithms

Correctness proofs

involved and hard to check

#### A zoo of automata



Algorithms

Correctness proofs

involved and hard to check

# Category Theory

Conceptual tools

Correctness proof(s)

Guidelines new algorithms

Unveil connections

# Category Theory

Conceptual tools

Correctness proof(s)

Guidelines new algorithms

Unveil connections

No free lunch!

#### Automata

$$X \rightarrow 2 \times X^A$$

**DFA** 

#### Automata

$$X \to 2 \times X^A$$

$$X \to \mathbb{R} \times (\mathbb{R}^X)^A$$

DFA

**WFA** 

#### Automata

$$X \to 2 \times X^A$$

$$X \to \mathbb{R} \times (\mathbb{R}^X)^A$$

**DFA** 

WFA

$$X \rightarrow FTX$$

Algebraic properties

**Transition structure** 

## $X \to FTX$

$$X o 2 imes X^A$$
 
$$X o \mathbb{R} imes (\mathbb{R}^X)^A$$
 DFA WFA

#### $X \rightarrow FTX$

$$X \to 2 \times X^A$$

$$X \to \mathbb{R} \times (\mathbb{R}^X)^A$$

**DFA** 

**WFA** 

 $2^{A^*}$ 

acceptance

 $\mathbb{R}^{A^*}$ 

**Vector space** 

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Language equivalence

equivalence

Weighted language equivalence **or** bisimilarity

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 $\mathbb{R}^{A^*}$  Vector space

Language equivalence

equivalence

Weighted language equivalence **or** bisimilarity

Proof methods for equivalence

# Up-to techniques

Algebraic structure



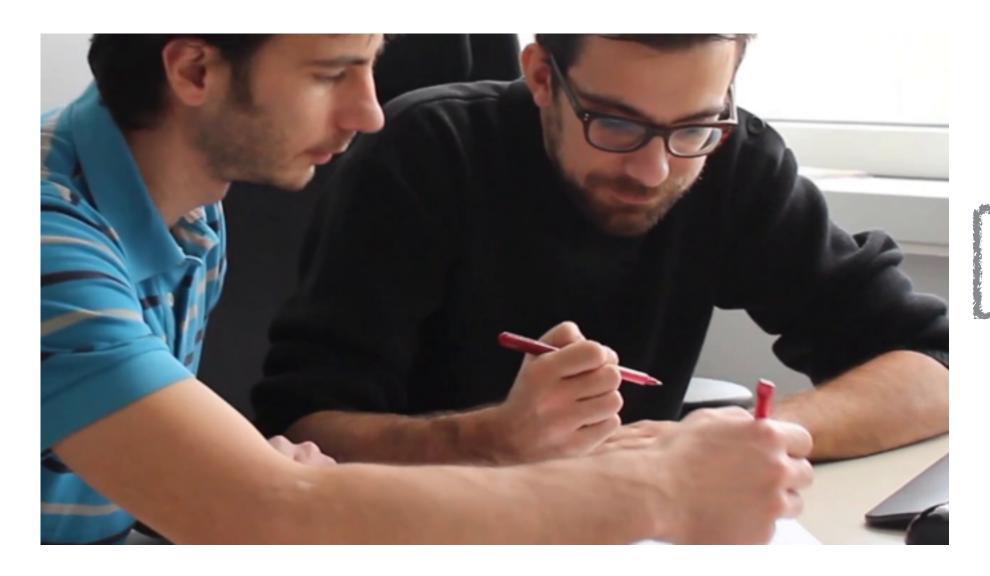
Better Proof Techniques

# Up-to techniques

Algebraic structure

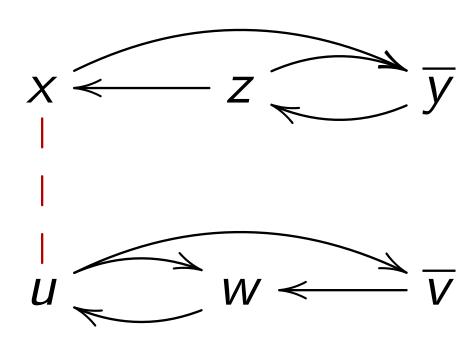


Better Proof Techniques

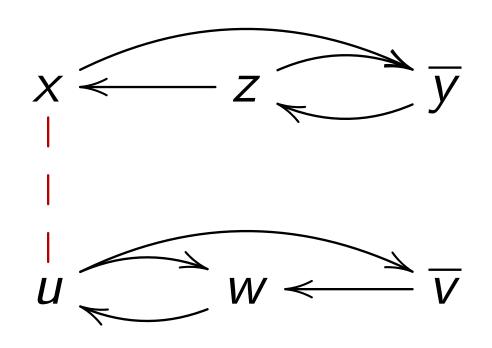


HKC algorithm - Bonchi and Pous 2014

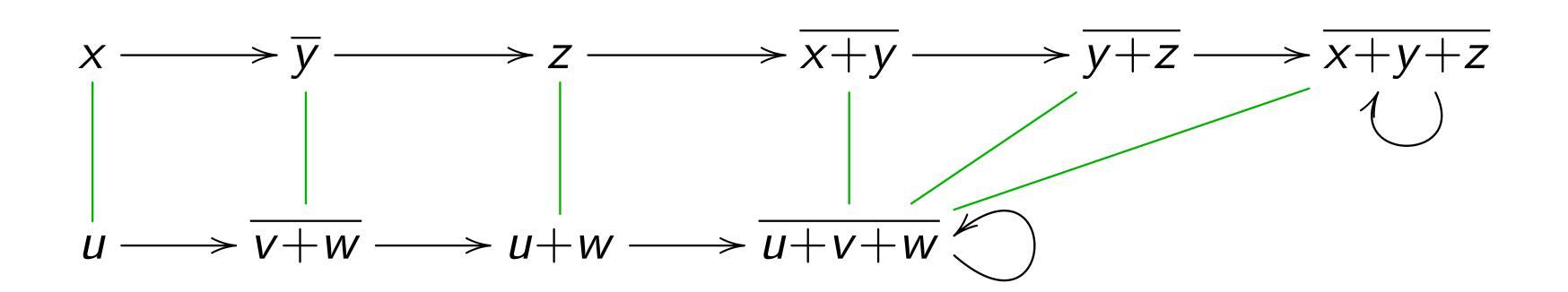
# Example



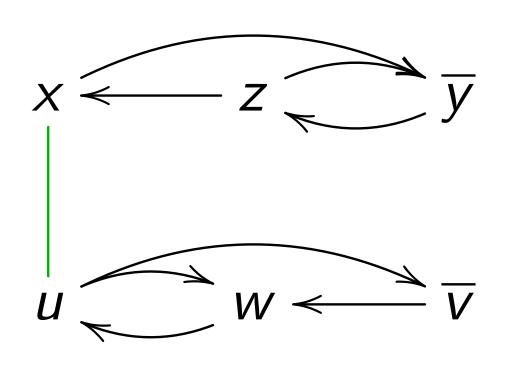
## Example



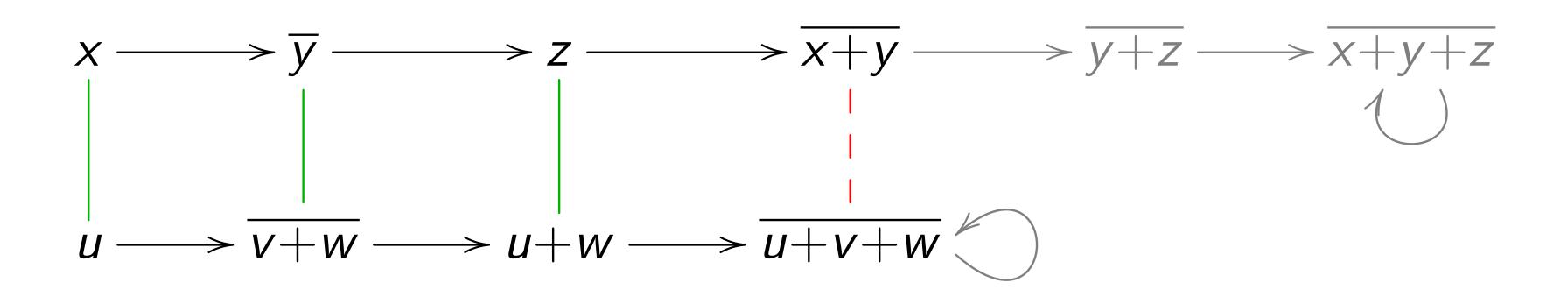
Build a bisimulation using powerset construction on the fly



## Example

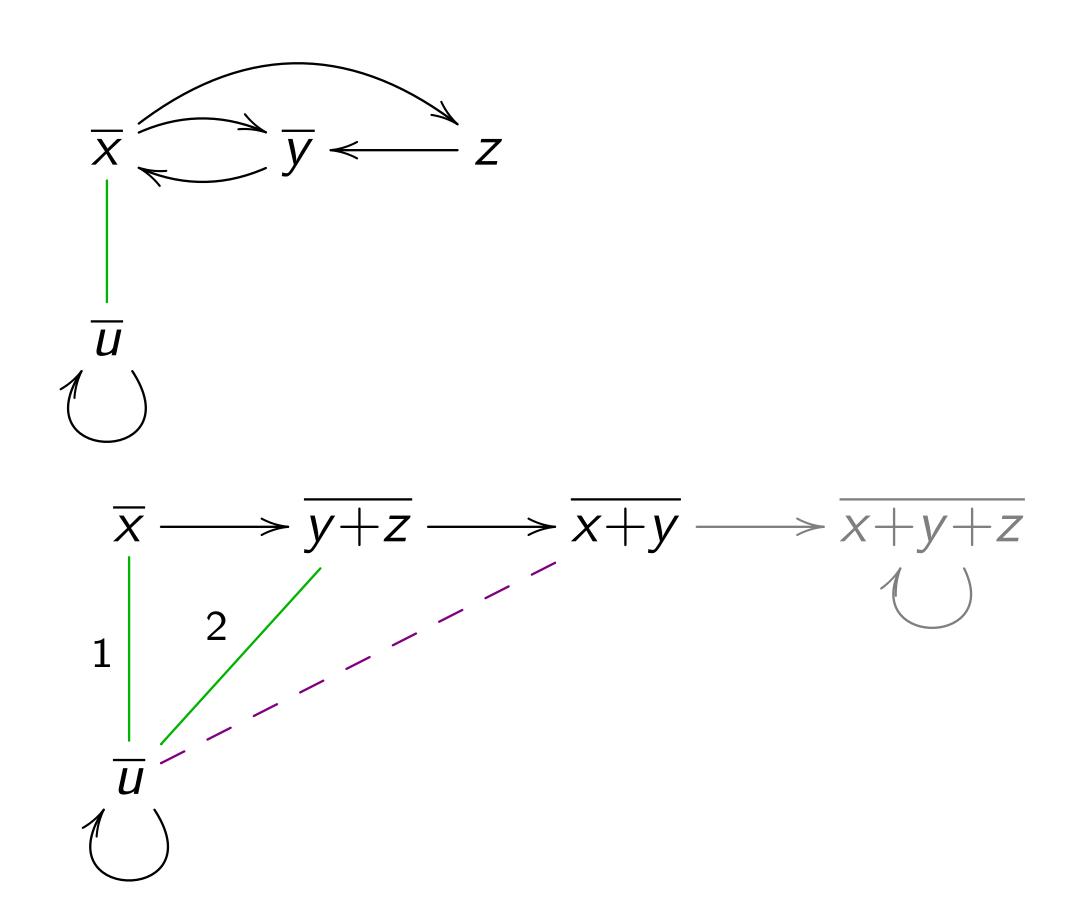


$$(x, u)$$
 $+ (y, v+w)$ 
 $= (x+y, u+v+w)$ 

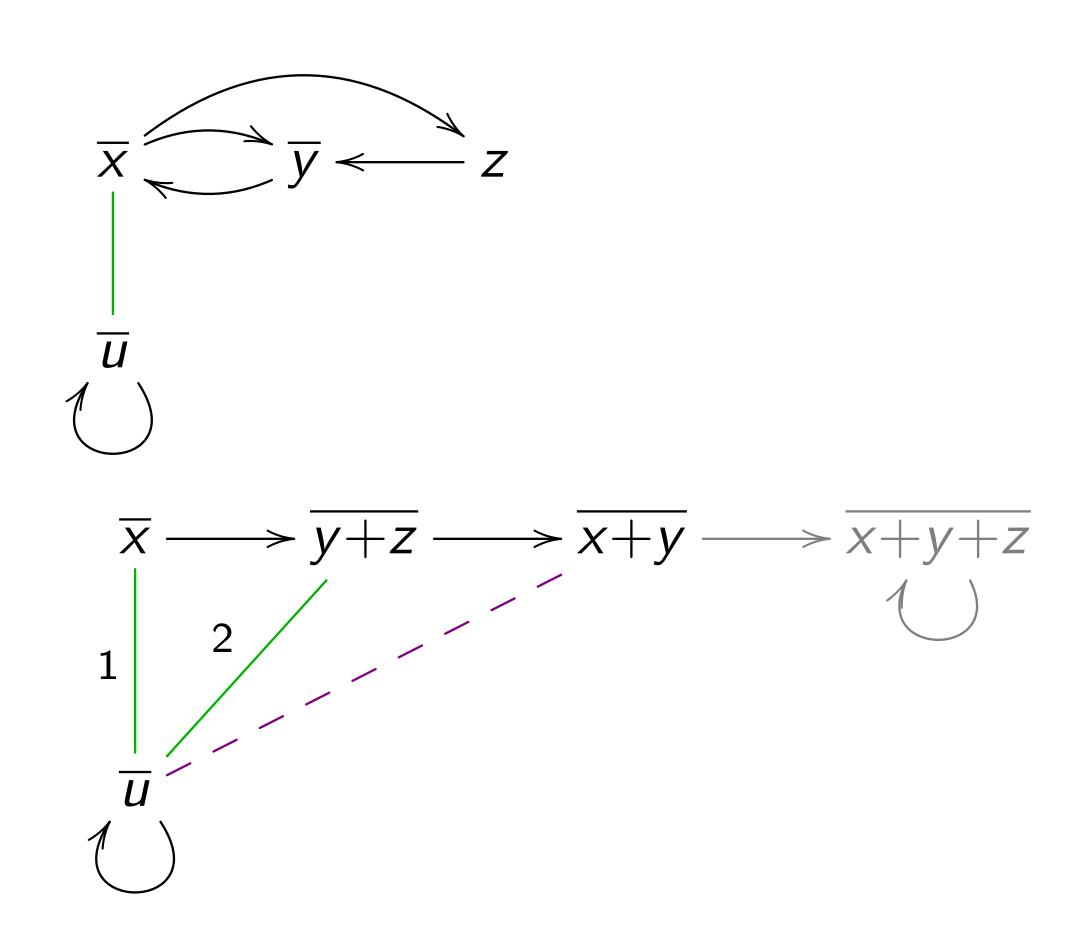


using bisimulations up to union

# Another example



## Another example

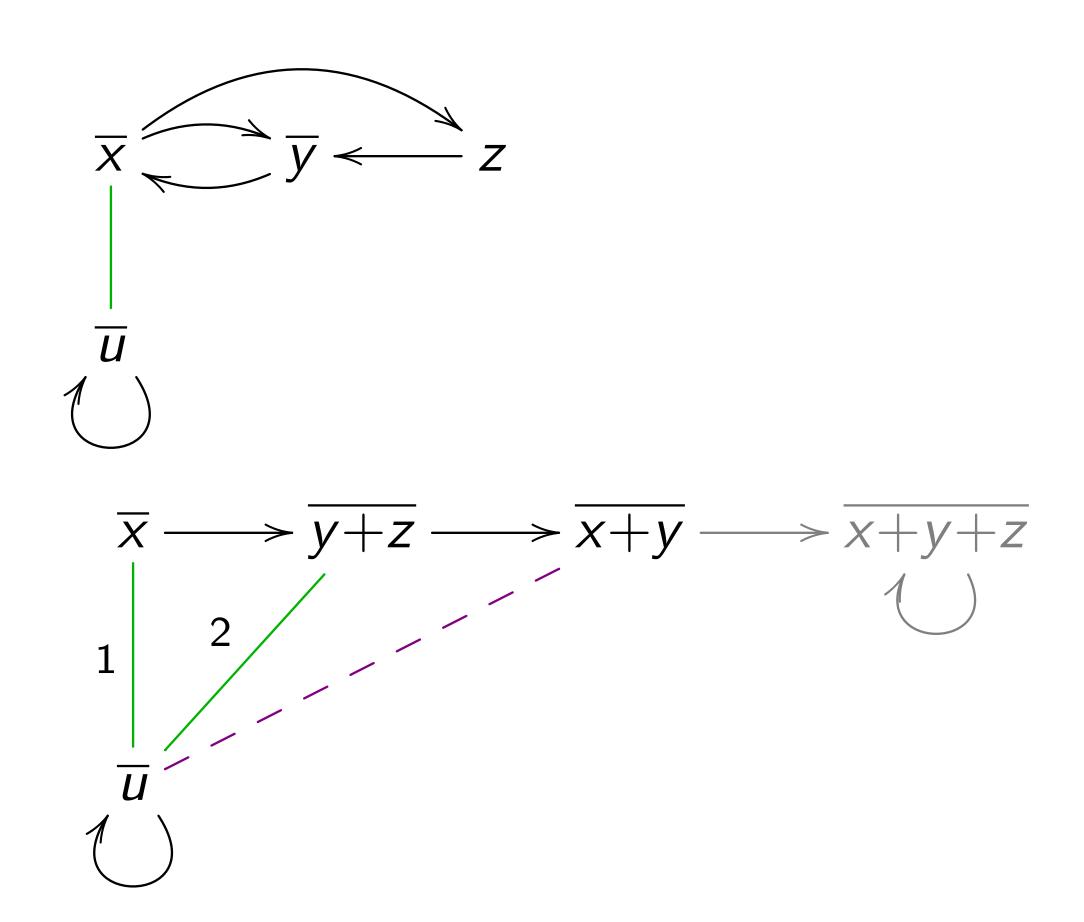


$$x+y = u+y \qquad (1)$$

$$= y+z+y \qquad (2)$$

$$= y+z \qquad (2)$$

## Another example



$$x+y = u+y \qquad (1)$$

$$= y+z+y \qquad (2)$$

$$= y+z \qquad (2)$$

$$= u \qquad (2)$$

Bisimulations up-to **congruence** HKC algorithm of Bonchi&Pous

# More examples

**Up-To Techniques for Weighted Systems. (TACAS '17)** 

Filippo Bonchi, Barbara König, Sebastian Küpper

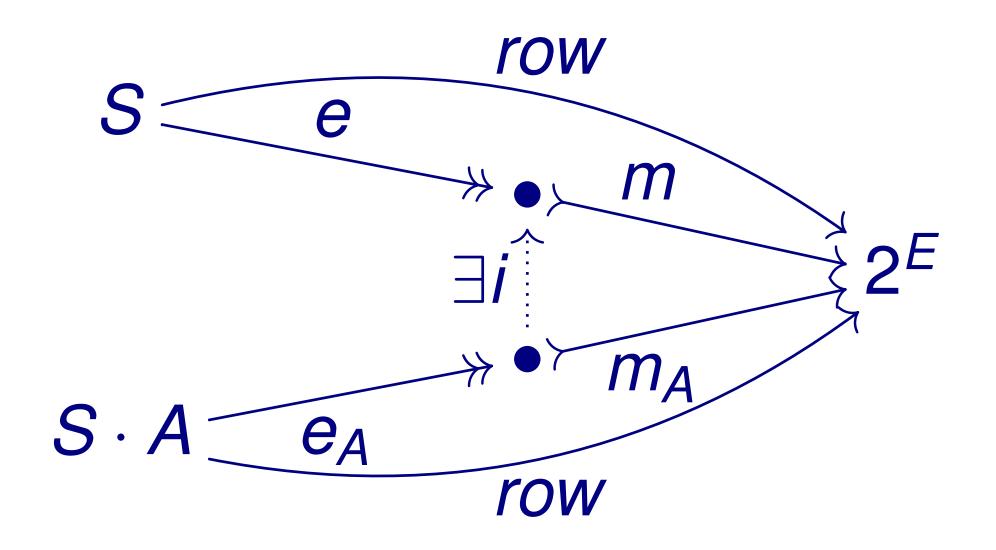
The Power of Convex Algebras (CONCUR' 17)

Filippo Bonchi, Alexandra Silva, Ana Sokolova

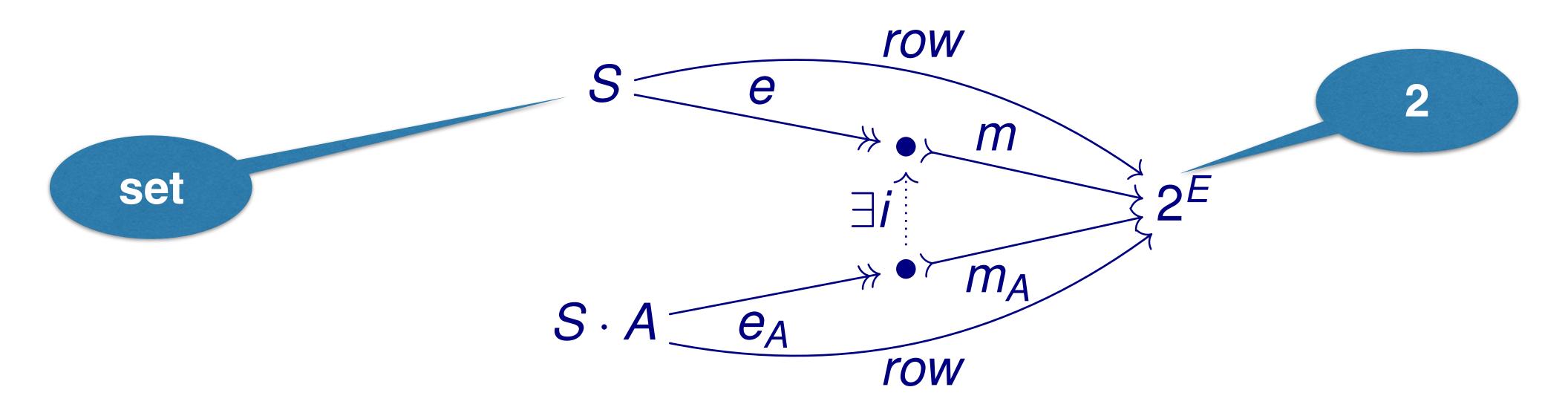
Coinduction up-to in a fibrational setting (CSL-LICS 2014)

Filippo Bonchi, Daniela Petrisan, Damien Pous, Jurriaan Rot

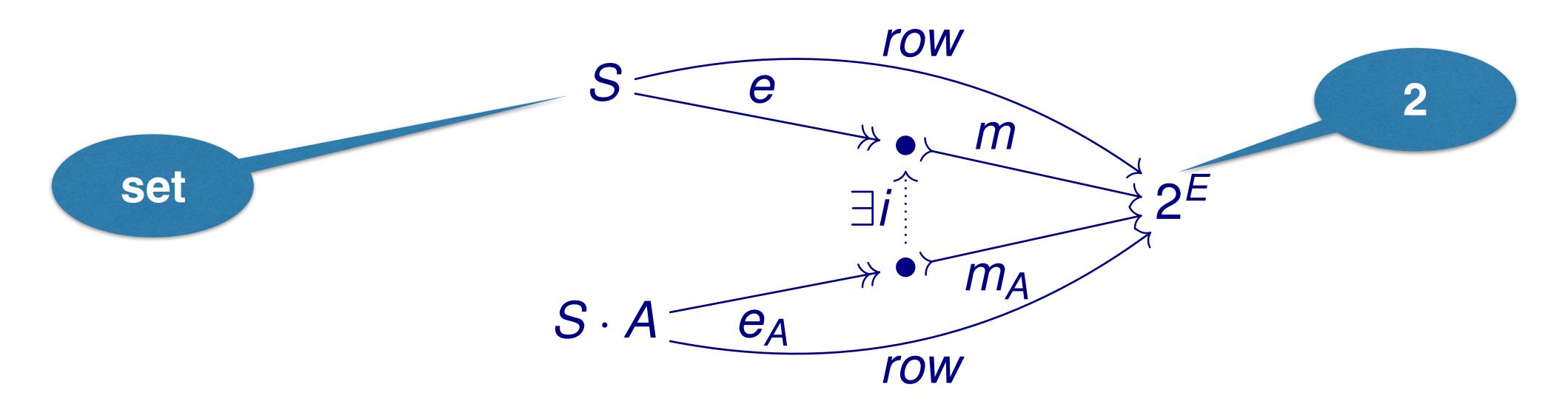
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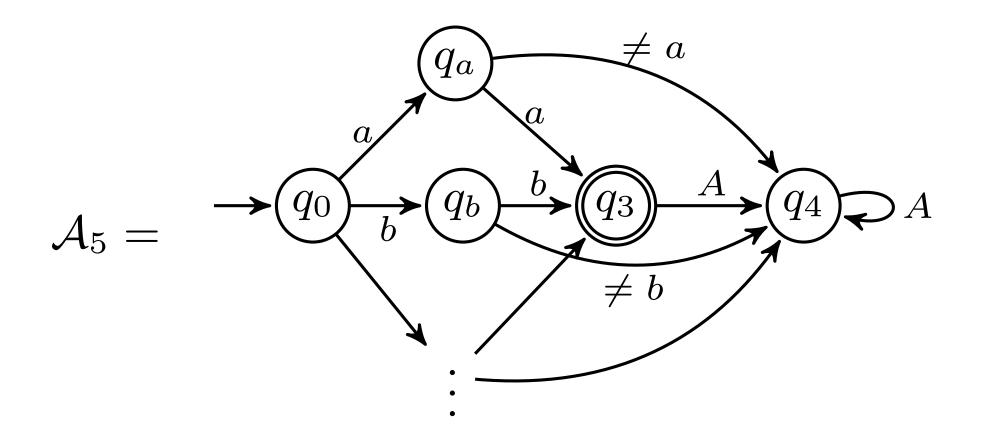
Can we develop L\* for infinite (nominal) sets?

# Infinite alphabets

$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

A infinite

$$\mathcal{L}_1 = \{aa, bb, cc, dd, \ldots\}$$



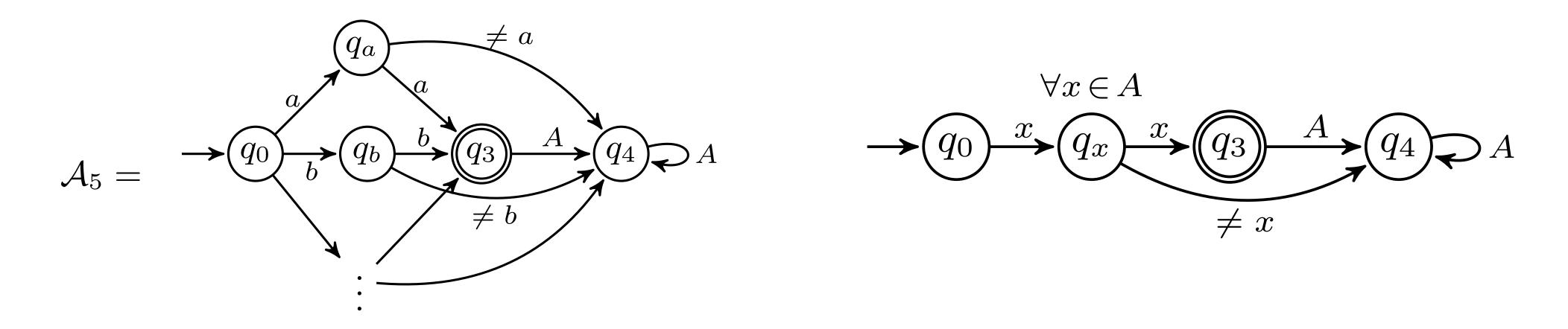
infinite automaton

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infinite automaton

but with a finite representation



Nominal sets





name binding alpha-equivalence

. . . . .



Nominal sets





name binding alpha-equivalence

. . . . .

Infinite sets



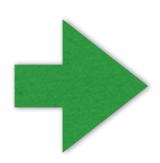
Nominal sets





name binding alpha-equivalence

Infinite sets with symmetries



Finitely representable



Nominal sets

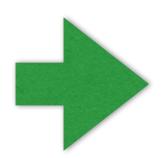




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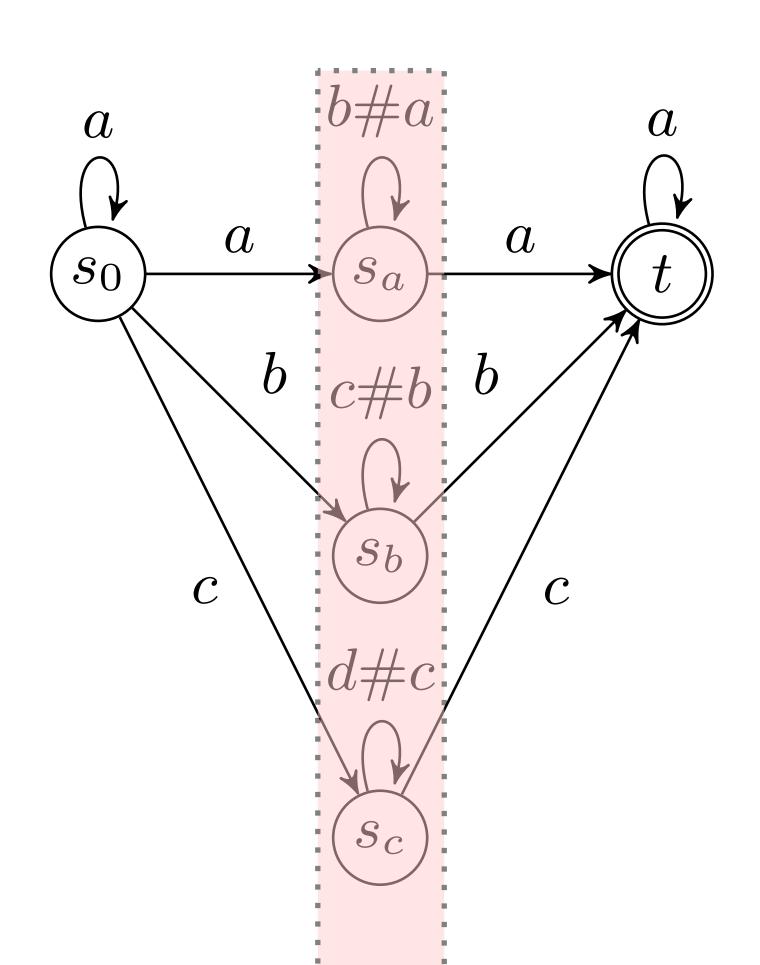






Automata theory over nominal sets

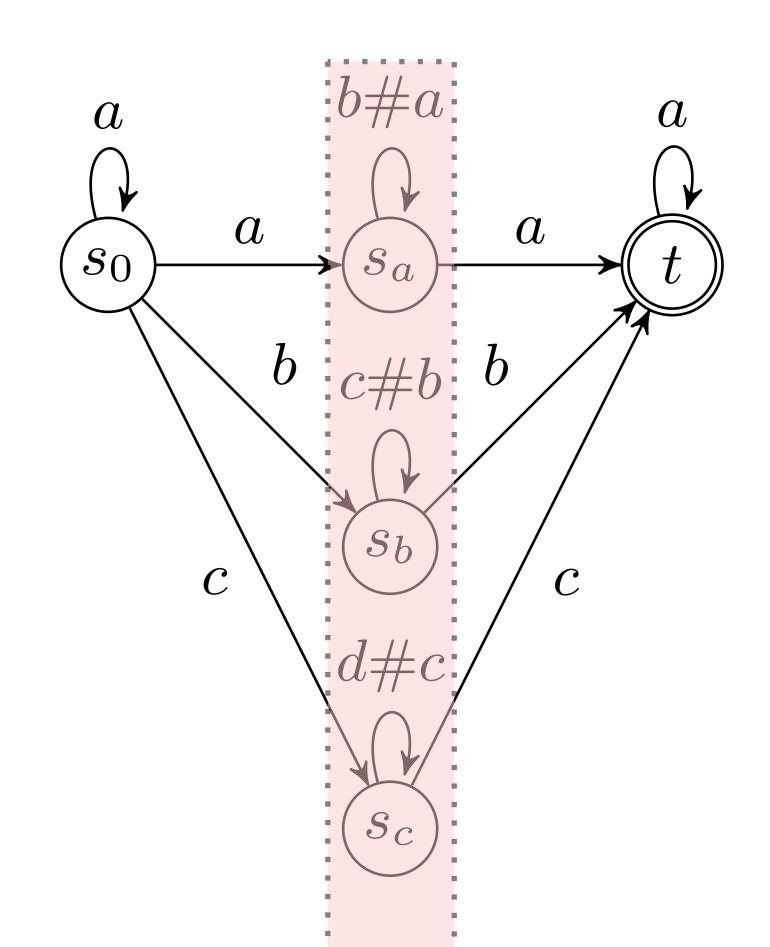
# Nominal automate



A infinite

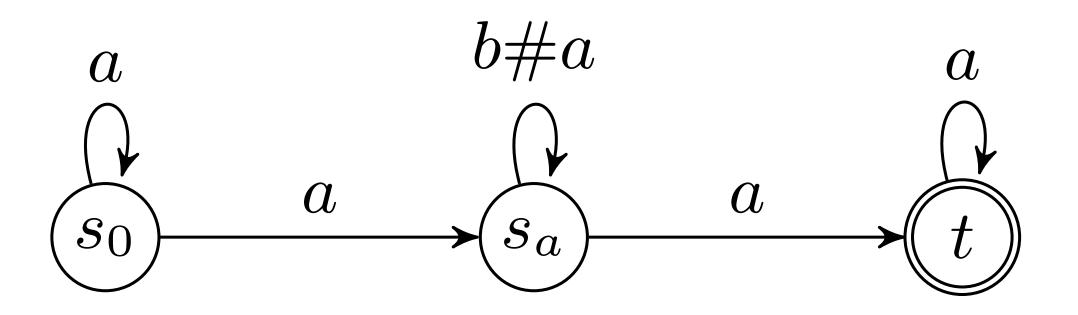
 $\{w \in \mathbb{A}^* \mid \exists a.a \text{ occurs twice in } w\}$ 

# Nominal automata



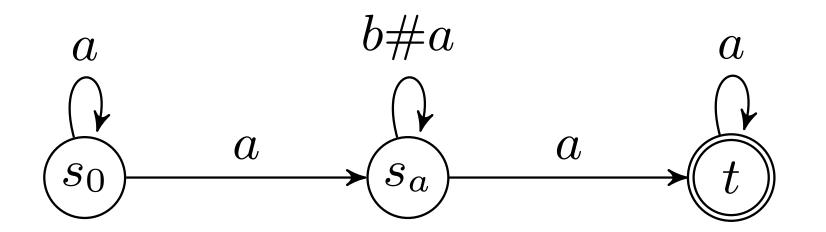
#### A infinite

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finite representation

# Nominal automata



finite representation

transition closed under permutations equivariant

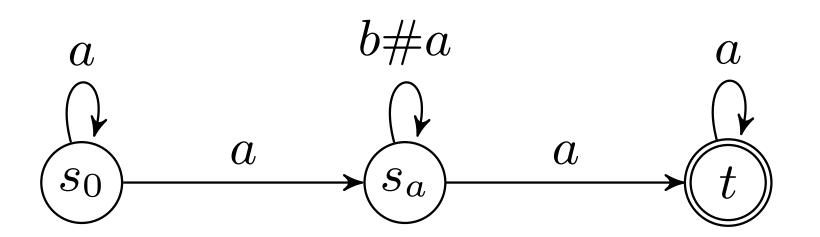
$$X = \{s_0\} + \mathbb{A} + \{t\}$$

$$\pi: \mathbb{A} \to \mathbb{A}$$

$$s_a \mapsto s_{\pi a}$$

$$s_a \xrightarrow{a} t \Rightarrow s_{\pi a} \xrightarrow{\pi a} t$$

# Nominal automate



finite representation

$$X = \{s_0\} + \mathbb{A} + \{t\}$$

canonical permutations

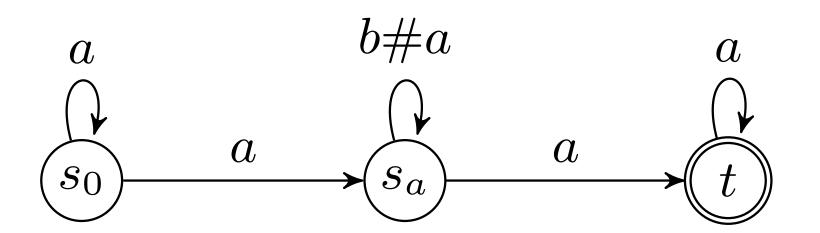
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**DFA in Nom** 

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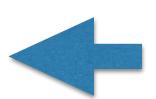
transition closed under permutations equivariant

$$s_a \xrightarrow{a} t \Rightarrow s_{\pi a} \xrightarrow{\pi a} t$$

algebraic structure

```
L* LEARNER
      S, E \leftarrow \{\epsilon\}
      repeat
           while (S, E) is not closed or not consistent
           if (S, E) is not closed
                 find s_1 \in S, a \in A such that
                      row(s_1a) \neq row(s), for all s \in S
                S \leftarrow S \cup \{s_1a\}
           if (S, E) is not consistent
                find s_1, s_2 \in S, a \in A, and e \in E such that
                      row(s_1) = row(s_2) and \mathcal{L}(s_1ae) \neq \mathcal{L}(s_2ae)
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           Make the conjecture M(S, E)
           if the Teacher replies \mathbf{no}, with a counter-example t
12
                 S \leftarrow S \cup \mathtt{prefixes}(t)
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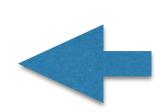


range over infinite sets

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range over infinite sets

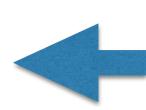


finding witnesses potentially requires checking infinite rows

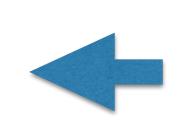
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range over infinite sets



finding witnesses potentially requires checking infinite rows



t has only finitely many prefixes, but an infinite S is necessary

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no finite automaton accepts

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```

**(P1)** the sets S, S·A and E admit a finite representation up to permutations; **(P2)** row is such that  $row(\pi(s))(\pi(e)) = row(s)(e)$ , for all  $s \in S$  and  $e \in E$ . Observation table admits a finite symbolic representation.

```
6' \quad S \leftarrow S \cup \text{orb}(sa)
9' \quad E \leftarrow E \cup \text{orb}(ae)
12' \quad E \leftarrow E \cup \text{prefixes}(\text{orb}(t))
```

only 3 lines changed!

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not really... all definitions have to be adapted to nominal/equivariant.

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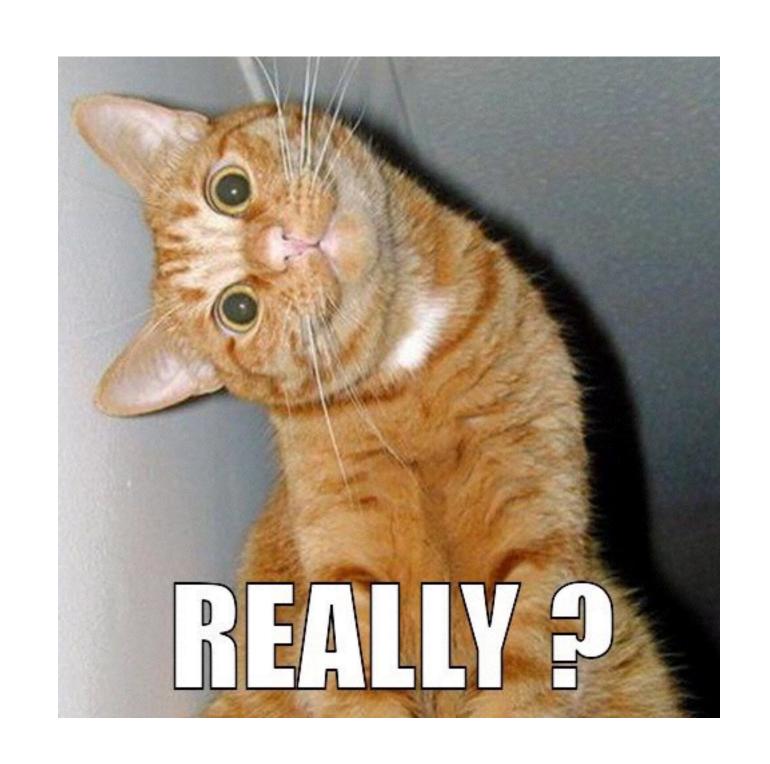
Correctness, termination, ... have to be re-proved!

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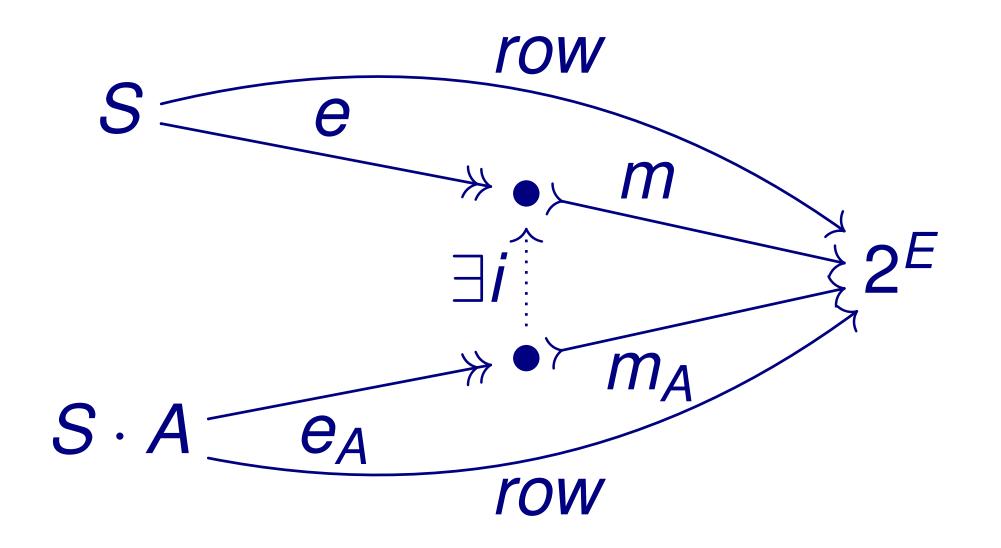
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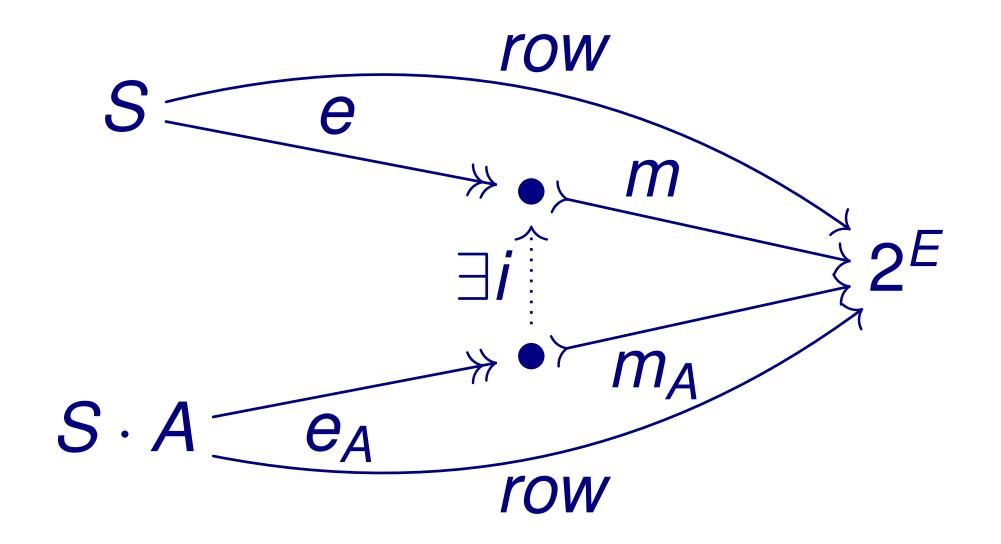
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(S, E, row) is *closed* if for all  $t \in S \cdot A$  there exists an  $s \in S$  such that row(t) = row(s).

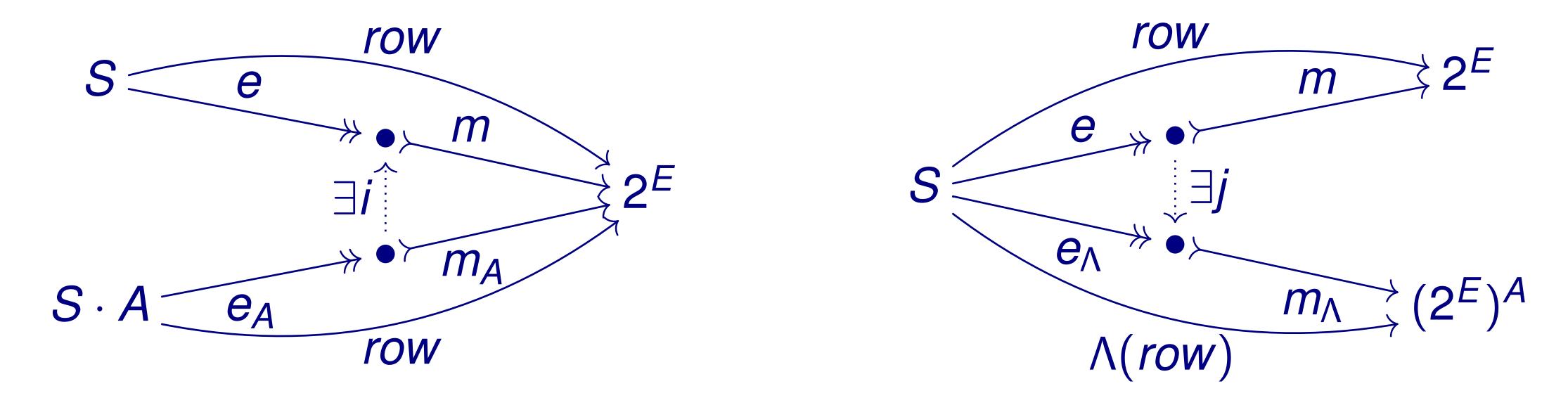


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(S, E, row) is *consistent* if whenever  $s_1, s_2 \in S$  are such that  $row(s_1) = row(s_2)$ , for all  $a \in A$ ,  $row(s_1a) = row(s_2a)$ .

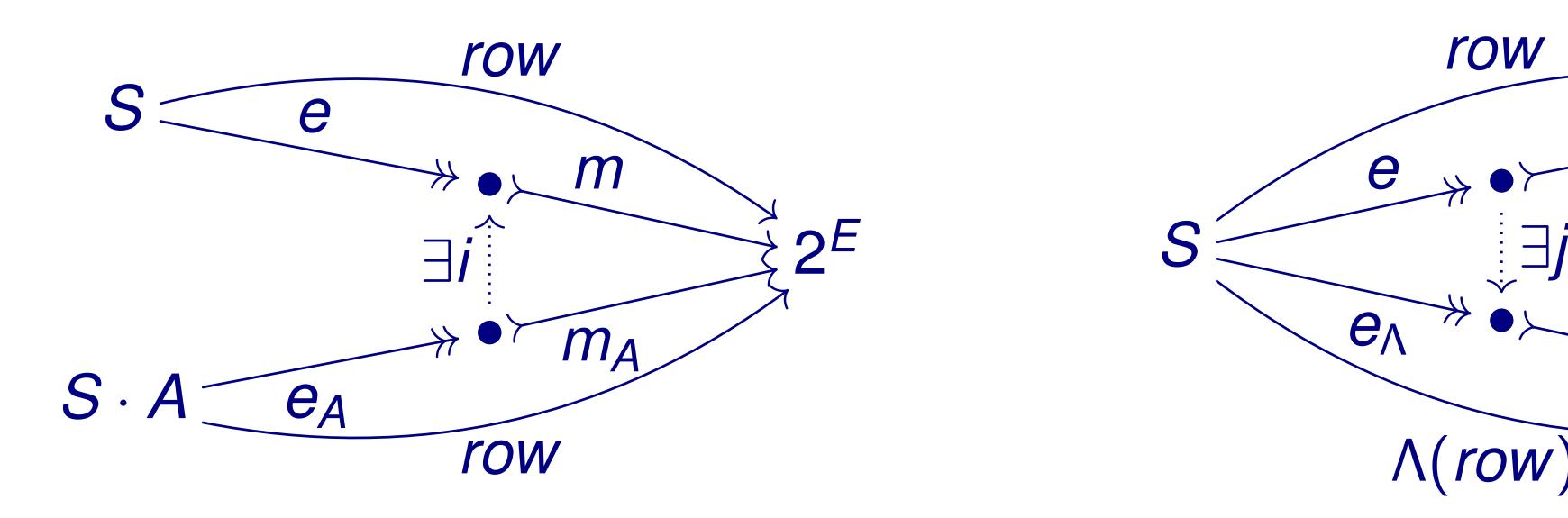


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m

 $m_{\wedge}$ 



(S, E, row) is *closed* if for all  $t \in S \cdot A$  there exists an  $s \in S$  such that row(t) = row(s).

(S, E, row) is c Pretty.... but is it useful? are such that  $row(s_1) = row(s_2)$ , for all  $a \in A$ ,  $row(s_1a) = row(s_2a)$ .

#### The power of abstraction

 $X \rightarrow 2 \times X^A$ 

**DFA in Nom** 

Definitions are the same

Proof of correctness is the same

#### The power of abstraction

 $X \rightarrow 2 \times X^A$ 

**DFA in Nom** 

Definitions are the same

Proof of correctness is the same

$$\begin{array}{c|c}
1 & init & final \\
A^* - - - - - & Q - - - - - + 2^{A*} \\
c & & \delta & \partial \\
(A^*)^A - - - - & Q^A - - - - + (2^{A*})^A
\end{array}$$

Category C = universe of state-spaces

Endofunctor  $F: \mathbb{C} \to \mathbb{C}$  = automaton type

$$FQ$$
 $\downarrow \delta_Q$ 
 $\downarrow I$ 
 $\downarrow Q$ 
 $\downarrow Q$ 
 $\downarrow Q$ 
 $\downarrow Y$ 

Category C = universe of state-spaces

Endofunctor  $F: \mathbb{C} \to \mathbb{C}$  = automaton type

$$C = Set$$

$$F = (-) \times A$$

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$$F = (-) \times A$$

$$Q imes A$$
  $\downarrow^{\delta_Q}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$   $\downarrow^{\Omega}$ 

Category C = universe of state-spaces

Endofunctor  $F: \mathbb{C} \to \mathbb{C}$  = automaton type

$$C = Set$$

$$F = (-) \times A$$

$$Q imes A$$
  $\downarrow \delta_Q$   $\downarrow \delta_Q$ 

Category C = universe of state-spaces

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$$F = (-) \times A$$

$$Q \times A$$
 
$$\downarrow^{\delta_Q}$$
 
$$\operatorname{init}_Q Q \operatorname{out}_Q$$
 
$$Y$$
 
$$q_0 \in Q$$

Category C = universe of state-spaces

Endofunctor  $F: \mathbb{C} \to \mathbb{C}$  = automaton type

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$$F = (-) \times A$$

$$\begin{array}{c}Q\times A\\ \downarrow \delta_Q\\ \operatorname{init}_Q Q \operatorname{out}_Q\\ \mathbf{1} \end{array}$$
 
$$q_0\in Q$$

Category C = universe of state-spaces

Endofunctor  $F: \mathbb{C} \to \mathbb{C}$  = automaton type

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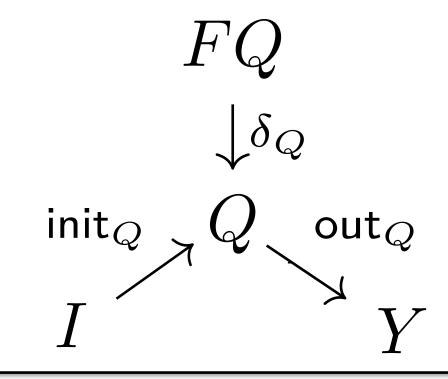
$$Q imes A$$
 
$$\downarrow^{\delta_Q}$$
 
$$\mathsf{init}_Q \qquad \mathsf{out}_Q$$
 
$$\mathbf{1} \qquad \mathbf{2}$$
 
$$q_0 \in Q \qquad F \subseteq Q$$

Abstract observation data structure

Abstract observation data structure

approximates

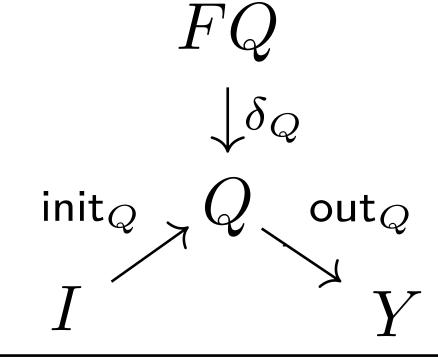


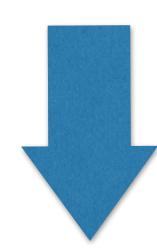


Abstract observation data structure

approximates

Target minimal automaton





abstract closedness and consistency

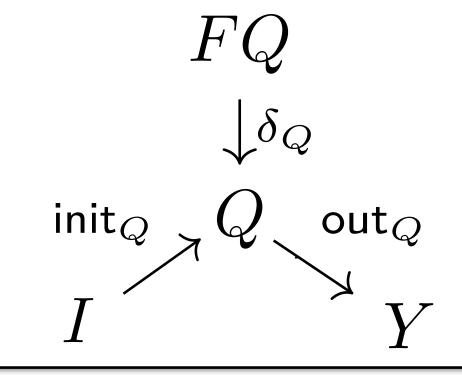
Hypothesis automaton

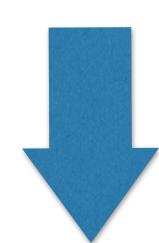
$$FH \\ \downarrow \delta_H \\ \text{init}_H \\ I \\ V$$

Abstract observation data structure

approximates

**Target minimal automaton** 





abstract closedness and consistency

Hypothesis automaton

$$FH \\ \downarrow \delta_H \\ \operatorname{init}_H \to H \\ I \\ Y$$

General correctness theorem

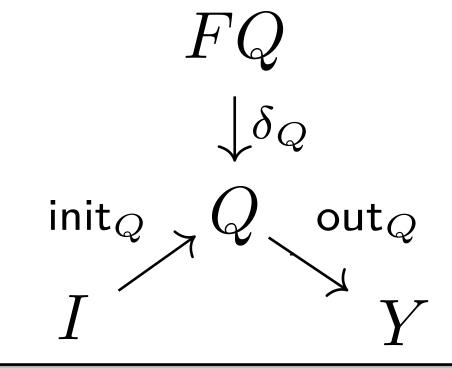
Guidelines for implementation

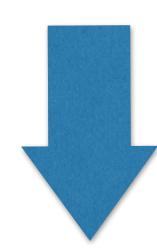
# Abstract learning

Abstract observation data structure

approximates

Target minimal automaton





abstract closedness and consistency

Hypothesis automaton

$$FH \\ \downarrow \delta_H \\ \operatorname{init}_H \to H \\ I \\ Y$$

General correctness theorem

Guidelines for implementation

CALF: Categorical Automata Learning Framework (arXiv:1704.05676)

Gerco van Heerdt, Matteo Sammartino, Alexandra Silva

#### Change base category

Set DFAs

Nom Nominal automata

Vect Weighted automata

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**Vect Weighted automata** 

#### Side-effects (via monads)

Powerset NFAs

Powerset with intersection Universal automata

Double powerset Alternating automata

#### Change base category

Change main data structure

Set DFAs

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**Vect Weighted automata** 

**Observation tables** 

**Discrimination trees** 

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**Learning Nominal Automata (POPL '17)** 

Set DFAs

Joshua Moerman, Matteo Sammartino, Alexandra Silva, Bartek Klin, Michal Szynwelski

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Learning Automata with Side-effects (arXiv:1704.08055)

Gerco van Heerdt, Matteo Sammartino, Alexandra Silva

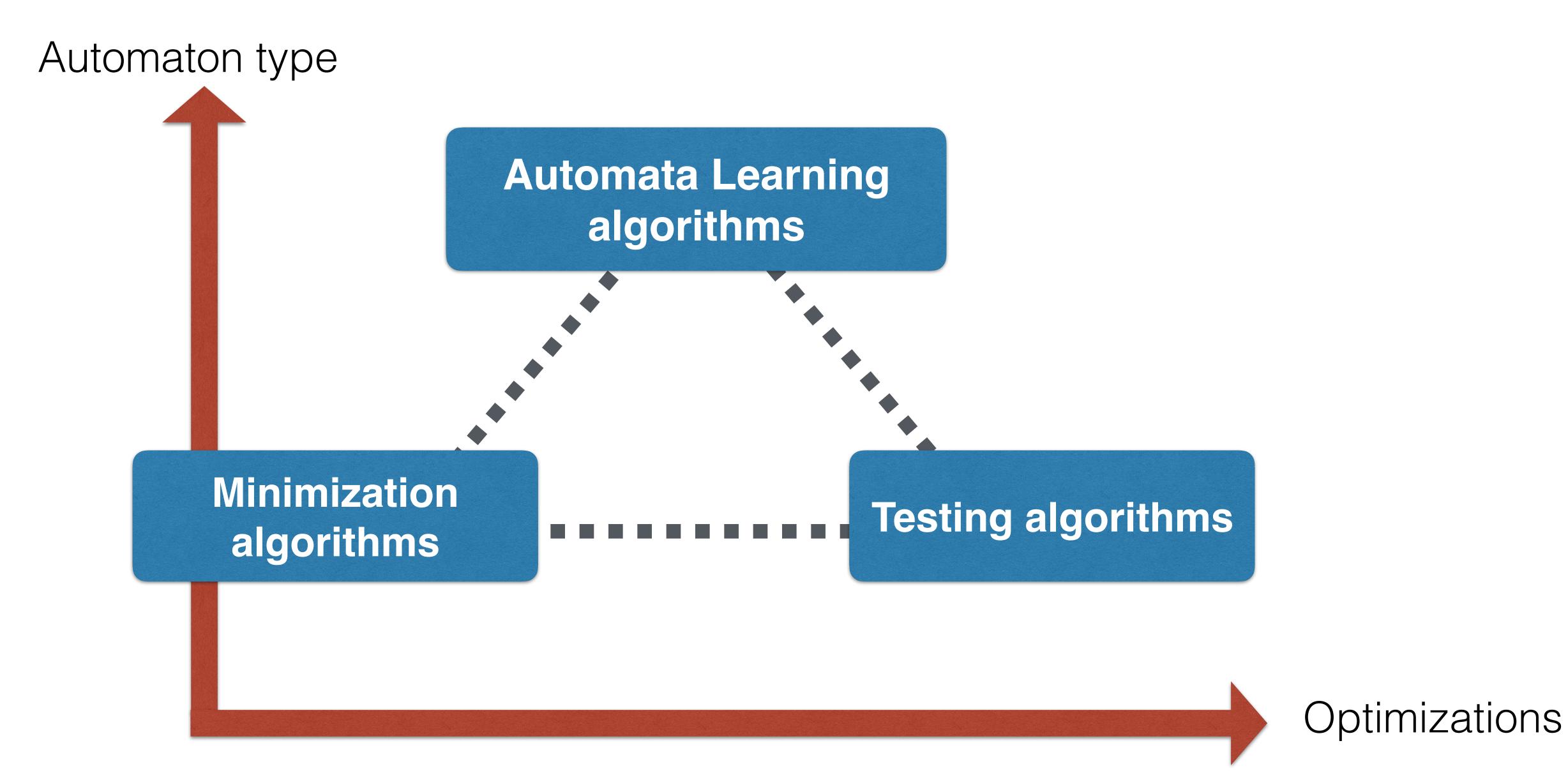
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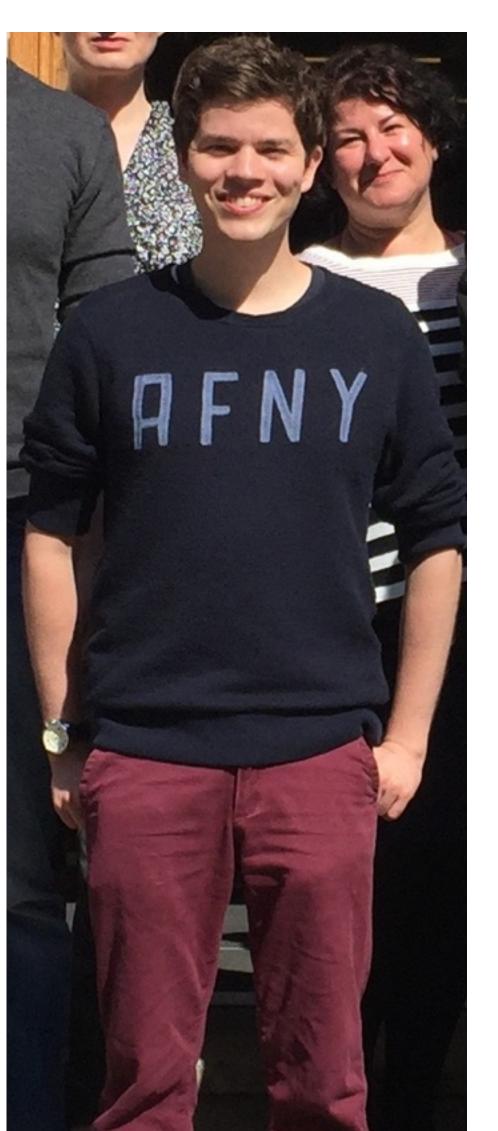
## Connections with other algorithms



# Ongoing and future work

- Library & tool to learn control + data-flow models (as nominal automata)
- Applications:
  - Specification mining
  - Network verification, with amazon
  - Verification of cryptographic protocols
  - Ransomware detection

# Ongoing and future work



Learning convex automata

Rich algebraic structure

Challenging analytical properties

### Conclusions

Category theory is a good playground to understand and generalise algorithms

### Conclusions

Category theory is a good playground to understand and generalise algorithms

Unveils connections and sets the scene

No free lunch



## Questions?

